Preprint typeset in JHEP style - HYPER VERSION

Note About Hamiltonian Formalism for General Non-Linear Massive Gravity Action in Stückelberg Formalism

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Abstract: In this note we try to prove the absence of the ghosts in case of the general non-linear massive gravity action in Stückelberg formalism. We argue that in order to find the explicit form of the Hamiltonian it is natural to start with the general non-linear massive gravity action found in arXiv:1106.3344 [hep-th]. We perform the complete Hamiltonian analysis of the Stückelberg form of the minimal the non-linear gravity action in this formulation and show that the constraint structure is so rich that it is possible to eliminate non-physical modes. Then we extend this analysis to the case of the general non-linear massive gravity action. We find the corresponding Hamiltonian and collection of the primary constraints. Unfortunately we are not able to finish the complete analysis of the stability of all constraints due to the complex form of one primary constraint so that we are not able to determine the conditions under which given constraint is preserved during the time evolution of the system.

Keywords: Massive Gravity.

Contents

1. Introduction and Summary

The first formulation of the massive gravity was performed by [1] at least in its linearized level as the propagation of the massive graviton above the flat background [1]. Even if the theory seems to be well defined at the linearized level there is a Boulware-Deser ghost [3] in the naive non-linear extension of the Fierz-Pauli formulation. On the other hand recently there has been much progress in the non-linear formulation of the massive gravity without the Boulware-Deser ghost [4, 5] and also [37, 38][2].

The Hamiltonian treatment of non-linear massive gravity theory was performed in many papers with emphasis on the general proof of the absence of the ghosts in given theory. The first attempt for the analysis of the constraint structure of the non-linear massive gravity was performed in [47]. However it turned out that this analysis was not complete and the wrong conclusion was reached as was then shown in the fundamental paper [40] where the complete Hamiltonian analysis of the gauge fixed form of the general non-linear massive gravity was performed. The fundamental result of given paper is the proof of the existence of two additional constraints in the theory which are crucial for the elimination of non physical modes and hence for the consistency of the non-linear massive gravity at least at the classical level.

Then the Hamiltonian analysis of the non-linear massive gravity in the Stückelberg formulation was performed in [48, 49]. Unfortunately the wrong conclusion was again reached in the first versions of given papers as was then shown in [50] where the absence of the ghosts in the minimal version of non-linear massive gravity was proven for the first time. Then an independent proof of the absence of the ghosts in the minimal version of non-linear massive gravity in the Stückelberg formulation was presented in [51].

However the proof of the absence of the ghosts in the general form of the non-linear massive gravity in the Stückelberg formulation is still lacking. The difficulty with the possible Hamiltonian formulation of given theory is that the action depends on the kinetic

[1]For recent review and extensive list of references, see [2].

[2]For related works, see [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 43].

terms of the Stückelberg fields in a highly non-linear way so that it seems to be impossible to find an explicit relation between canonical conjugate momenta and the time derivatives of the Stückelberg fields. On the other hand there exists the formulation of the non-linear massive gravity action with the linear dependence on the kinetic term for the Stückelberg fields. This is the form of the non-linear massive gravity action that arises from the original one when the redefinition of the shift functions is performed [38, 37, 41, 42]. The goal of this paper is to perform the Hamiltonian analysis of given action and try to identify all constraints.

As we argued above the main advantage of the formulation of the non-linear massive gravity with redefined shift function is that the kinetic term for the Stückelberg fields appears linearly and hence it is easy to find the corresponding Hamiltonian even for the general form of the non-linear massive gravity action. Further it is possible to identify four primary constraints of the theory where the three ones are parts of the generators of the spatial diffeomorphism. Note that the presence of the diffeomorphism constraints is the reflection of the fact that we have manifestly diffeomorphism invariant theory. On the other hand the fourth primary constraint could be responsible for the elimination of the additional non physical mode. However this claim is only true when the requirement of the preservation of given constraint during the time development of the system generates another additional constraint. Unfortunately we find that the original primary constraint cannot provide such additional constraint due to the fact that the Poisson bracket between the primary constraints defined at different space points is non zero. For that reason we should find another constraint that obeys the property that Poisson bracket between these constraints defined at different space points is zero. We find such a constraint in the case of the minimal non-linear massive gravity action and we show that this constraint has the same form as the primary constraint found in [51]. Then we will be able to show that the requirement of the preservation of given constraint during the time evolution of the system implies the additional constraint and these two constraints together allow to eliminate two non-physical modes. This result agrees with previous two independent analysis performed in [50] and in [51].

Unfortunately we are not able to reach the main goal of this paper which is the proof of the absence of the ghosts for the general non-linear massive gravity theory in Stückelberg formalism. The reason is that we are not able to find the primary constraint that has vanishing Poisson bracket between these constraints defined at different space points and that has the Poisson brackets with another constraints that vanish on the constraint surface. This is crucial condition for the existence of the additional constraint. It is rather worrying that we are not able to finish the Hamiltonian analysis for the general non-linear massive gravity action especially in the light of the very nice proof of the absence of the ghosts in case of the gauge fixed non-linear massive gravity action [40]. However there is a possibility that the proof of the absence of the ghosts for general non-linear massive gravity action in Stückelberg formalism could be found in the very elegant formulation of the massive and multi metric theories of gravity presented in [23]. We hope to return to this problem in future.

The structure of this paper is as follows. In the next section (2) we introduce the

non-linear massive gravity action and perform the field redefinition of the shift function. We also we find the Hamiltonian formulation of the minimal form of the non-lineal massive gravity in Stückelberg formulation with redefined shift functions and we find that given theory is free from the ghosts. Then we extend this approach to the case of the general non-linear massive gravity theory in the section (**??**). We identify all primary constraints and discuss the difficulties that prevent us to finish the complete Hamiltonian analysis.

2. Non-linear Massive Gravity with Redefined Shift Functions

As we stressed in the introduction section the goal of this paper is to perform the Hamiltonian analysis of the general non-linear massive gravity with presence of the Stückelberg fields. It turns out that it is useful to consider this action with redefined shift functions [38, 37, 41, 42]. More explicitly, let us begin with following general form of the non-linear massive gravity action

$$S = M_p^2 \int d^4x \sqrt{-\hat{g}}[{}^{(4)}R + 2m^2 \sum_{n=0}^{3} \beta_n e_n(\sqrt{\hat{g}^{-1}f})] \ , \tag{2.1}$$

where $e_k(\mathbf{A})$ are elementary symmetric polynomials of the eigenvalues of $\mathbf{A}$. For generic 4×4 matrix they are given by

$$\begin{aligned}
e_0(\mathbf{A}) &= 1 \ , \\
e_1(\mathbf{A}) &= [\mathbf{A}] \ , \\
e_2(\mathbf{A}) &= \frac{1}{2}([\mathbf{A}]^2 - [\mathbf{A}^2]) \ , \\
e_3(\mathbf{A}) &= \frac{1}{6}\left([\mathbf{A}]^3 - 3[\mathbf{A}][\mathbf{A}^2] + 2[\mathbf{A}^3]\right) \ , \\
e_4(\mathbf{A}) &= \frac{1}{24}\left([\mathbf{A}]^4 - 6[\mathbf{A}]^2[\mathbf{A}^2] + 3[\mathbf{A}^2]^2 + 8[\mathbf{A}][\mathbf{A}^3] - 6[\mathbf{A}^4]\right) \ , \\
e_k(\mathbf{A}) &= 0 \ , \ \text{for } k > 4 \ ,
\end{aligned} \tag{2.2}$$

where $\mathbf{A}^\mu_{\ \nu}$ is 4×4 matrix and where

$$[\mathbf{A}] = \mathrm{Tr}\mathbf{A}^\mu_{\ \mu} \ . \tag{2.3}$$

Of the four β_n two combinations are related to the mass and the cosmological constant while the remaining two combinations are free parameters. If we consider the case when the cosmological constant is zero and the parameter m is mass, the four β_n are parameterized in terms of the α_3 and α_4 as [5]

$$\beta_n = (-1)^n \left(\frac{1}{2}(4-n)(3-n) - (4-n)\alpha_3 + \alpha_4\right) \ . \tag{2.4}$$

The minimal action corresponds to $\beta_2 = \beta_3 = 0$ that implies $\alpha_3 = \alpha_4 = 1$ and consequently $\beta_0 = 3$, $\beta_1 = -1$.

We consider the massive gravity with that is manifestly diffeomorphism invariant. This can be ensured with the help of 4 scalar fields ϕ^A , $A = 0,1,2,3$ so that

$$\hat{g}^{\mu\nu} f_{\nu\rho} = \hat{g}^{\mu\nu} \partial_\nu \phi^A \partial_\rho \phi_A \ . \tag{2.5}$$

Then we have

$$N^2 \hat{g}^{-1} f = \begin{pmatrix} -f_{00} + N^l f_{l0} & -f_{0j} + N^l f_{lj} \\ N^2 g^{il} f_{l0} - N^i(-f_{00} + N^l f_{l0}) & N^2 g^{il} f_{lj} - N^i(-f_{0j} + N^l f_{lj}) \end{pmatrix} \ , \tag{2.6}$$

where we also used $3+1$ decomposition of the four dimensional metric $\hat{g}_{\mu\nu}$ [45, 46]

$$\begin{aligned} &\hat{g}_{00} = -N^2 + N_i g^{ij} N_j \ , \quad \hat{g}_{0i} = N_i \ , \quad \hat{g}_{ij} = g_{ij} \ , \\ &\hat{g}^{00} = -\frac{1}{N^2} \ , \quad \hat{g}^{0i} = \frac{N^i}{N^2} \ , \quad \hat{g}^{ij} = g^{ij} - \frac{N^i N^j}{N^2} \ . \end{aligned} \tag{2.7}$$

Let us now perform the redefinition of the shift function N^i that was introduced in [37, 38, 41, 42]

$$N^i = M \tilde{n}^i + f^{ik} f_{0k} + N \tilde{D}^i{}_j \tilde{n}^j \ , \tag{2.8}$$

where

$$\tilde{x} = 1 - \tilde{n}^i f_{ij} \tilde{n}^j \ , \quad M^2 = -f_{00} + f_{0k} f^{kl} f_{l0} \tag{2.9}$$

and where we defined f^{ij} as the inverse to f_{ij} in the sense [3]

$$f_{ik} f^{kj} = \delta_i^{\ j} \ . \tag{2.10}$$

Finally note that the matrix $\tilde{D}^i{}_j$ obeys the equation

$$\sqrt{\tilde{x}} \tilde{D}^i{}_j = \sqrt{(g^{ik} - \tilde{D}^i{}_m \tilde{n}^m \tilde{D}^k{}_n \tilde{n}^n) f_{kj}} \tag{2.11}$$

and also following important identity

$$f_{ik} \tilde{D}^k{}_j = f_{jk} \tilde{D}^k{}_i \ . \tag{2.12}$$

Let us now concentrate on the minimal form of the non-linear massive gravity action. Using the redefinition (2.8) we find that it takes the form

$$S = M_p^2 \int d^3\mathbf{x} dt [N\sqrt{g} \tilde{K}_{ij} \mathcal{G}^{ijkl} \tilde{K}_{kl} + N\sqrt{g} R - \sqrt{g} M U - 2m^2 (N\sqrt{g}\sqrt{\tilde{x}} D^i{}_i - 3N\sqrt{g})] \ , \tag{2.13}$$

where

$$U = 2m^2 \sqrt{\tilde{x}} \ , \tag{2.14}$$

[3] Note that in our convention f^{ik} coincides with $({}^3f^{-1})^{ik}$ presented in [41, 42, 38, 37].

and where we used the $3+1$ decomposition of the four dimensional scalar curvature

$$ {}^{(4)}R = \tilde{K}_{ij}\mathcal{G}^{ijkl}\tilde{K}_{kl} + R \ , \tag{2.15} $$

where R is three dimensional scalar curvature and where

$$ \mathcal{G}^{ijkl} = \frac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - g^{ij}g^{kl} \tag{2.16} $$

with inverse

$$ \mathcal{G}_{ijkl} = \frac{1}{2}(g_{ik}g_{jl} + g_{il}g_{jk}) - \frac{1}{2}g_{ij}g_{kl} \ , \quad \mathcal{G}_{ijkl}\mathcal{G}^{klmn} = \frac{1}{2}(\delta_i^m\delta_j^n + \delta_i^n\delta_j^m) \ . \tag{2.17} $$

Note that in (2.15) we ignored the terms containing total derivatives. Finally note that $\tilde{K}_{ij}$ is defined as

$$ \tilde{K}_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j(\tilde{n}, g) - \nabla_j N_i(\tilde{n}, g)) \ , \tag{2.18} $$

where N_i depends on $\tilde{n}^i$ and g through the relation (2.8).

At this point we should stress the reason why we consider the non-linear massive gravity action in the form (2.13). The reason is that we want to perform the Hamiltonian analysis for the general non-linear massive gravity action written in the Stückelberg formalism. It turns out that the action (2.13) has formally the same form as in case of the general non-linear massive gravity action when we replace U with more general form whose explicit form was determined in [37, 38, 41, 42]. On the other hand the main advantage of the action (2.13) is that it depends on the time derivatives of ϕ^A through the term M and that this term appears linearly in the action (2.13). We should compare this fact with the original form of the non-linear massive gravity action where the dependence on the time derivatives of ϕ^A is highly non-linear and hence it is very difficult to find corresponding Hamiltonian.

Explicitly, from (2.13) we find the momenta conjugate to $N, \tilde{n}^i$ and g_{ij}

$$ \pi_N \approx 0 \ , \pi_i \approx 0 \ , \pi^{ij} = M_p^2\sqrt{g}\mathcal{G}^{ijkl}\tilde{K}_{kl} \tag{2.19} $$

and the momentum conjugate to ϕ^A

$$ p_A = -\left(\frac{\delta M}{\delta\partial_t\phi^A}\tilde{n}^i + f^{ij}\partial_j\phi_A\right)\mathcal{R}_i - M_p^2\sqrt{g}\frac{\delta M}{\partial_t\phi^A}U \ , \tag{2.20} $$

where

$$ \mathcal{R}_i = -2g_{ik}\nabla_j\pi^{kj} \ . \tag{2.21} $$

It turns out that it is useful to write M^2 in the form

$$ M^2 = -\partial_t\phi^A\mathcal{M}_{AB}\partial_t\phi^B \ , \quad \mathcal{M}_{AB} = \eta_{AB} - \partial_i\phi_A f^{ij}\partial_j\phi_B \ , \tag{2.22} $$

where by definition the matrix $\mathcal{M}_{AB}$ obeys following relations

$$\mathcal{M}_{AB}\eta^{BC}\mathcal{M}_{CD} = \mathcal{M}_{AD} \ , \quad \det \mathcal{M}^A_{\ B} = 1 \tag{2.23}$$

together with

$$\partial_i\phi^A\mathcal{M}_{AB} = \partial_i\phi_B - \partial_i\phi^A\partial_k\phi_A f^{kl}\partial_l\phi_B = 0 \ . \tag{2.24}$$

With the help of these results we find

$$p_A + \mathcal{R}_i f^{ij}\partial_j\phi_A = (\tilde{n}^i\mathcal{R}_i + M_p^2\sqrt{g}U)\frac{1}{M}\mathcal{M}_{AB}\partial_t\phi^B \tag{2.25}$$

and consequently

$$M^2 = -\partial_t\phi^A\mathcal{M}_{AB}\partial_t\phi^B = -\frac{M^2}{(\tilde{n}^i\mathcal{R}_i + M_p^2\sqrt{g}U)^2}(p_A + \mathcal{R}_i f^{ij}\partial_j\phi_A)\eta^{AB}(p_B + \mathcal{R}_i f^{ij}\partial_j\phi_B) \tag{2.26}$$

which however implies following primary constraint

$$\Sigma_p = (\tilde{n}^i\mathcal{R}_i + M_p^2\sqrt{g}U)^2 + (p_A + \mathcal{R}_i f^{ij}\partial_j\phi_A)(p^A + \mathcal{R}_i f^{ij}\partial_j\phi^A) \approx 0 \ . \tag{2.27}$$

Note that using (2.24) we obtain another set of the primary constraints

$$\partial_i\phi^A\Pi_A = \partial_i\phi^A p_A + \mathcal{R}_i = \Sigma_i \approx 0 \ . \tag{2.28}$$

Observe that using (2.28) we can write

$$p_A + \mathcal{R}_i f^{ij}\partial_j\phi_A = \mathcal{M}_{AC}\eta^{CB}p_B + \Sigma_i f^{ij}\partial_j\phi_A \tag{2.29}$$

so that we can rewrite Σ_p into the form

$$\Sigma_p = (\tilde{n}^i\mathcal{R}_i + M_p^2\sqrt{g}U)^2 + p_A\mathcal{M}^{AB}p_B + H^i\Sigma_i \ , \tag{2.30}$$

where H^i are functions of the phase space variables. As a result we see that it is natural to consider following independent constraint Σ_p

$$\Sigma_p = (\tilde{n}^i\mathcal{R}_i + M_p^2\sqrt{g}U)^2 + p_A\mathcal{M}^{AB}p_B \approx 0 \ . \tag{2.31}$$

We return to the analysis of the constraint Σ_p below.

Now we are ready to write the extended Hamiltonian which includes all the primary constraints

$$H_E = \int d^3\mathbf{x}(N\mathcal{C}_0 + v_N\pi_N + v^i\pi_i + \Omega_p\Sigma_p + \Omega^i\tilde{\Sigma}_i) \ , \tag{2.32}$$

where

$$\mathcal{C}_0 = \frac{1}{\sqrt{g}M_p^2}\pi^{ij}\mathcal{G}_{ijkl}\pi^{kl} - M_p^2\sqrt{g}R + 2m^2M_p^2\sqrt{g}\sqrt{\tilde{x}}\tilde{D}^i_{\ i} - 6m^2M_p^2\sqrt{g} + \tilde{D}^i_{\ j}\tilde{n}^j\mathcal{R}_i \tag{2.33}$$

and where we introduced the constraints $\tilde{\Sigma}_i$ defined as

$$\tilde{\Sigma}_i = \Sigma_i + \partial_i \tilde{n}^i \pi_i + \partial_j(\tilde{n}^j \pi_i) \ . \tag{2.34}$$

Note that $\tilde{\Sigma}_i$ is defined as linear combination of the constraints $\Sigma_i \approx 0$ together with the constraints $\pi_i \approx 0$.

To proceed further we have to check the stability of all constraints. To do this we have to calculate the Poisson brackets between all constraints and the Hamiltonian H_E. Note that we have following set of the canonical variables $g_{ij}, \pi^{ij}, \phi^A, p_A, \tilde{n}^i, \pi_i$ and N, π_N with non-zero Poisson brackets

$$\begin{aligned}
\left\{g_{ij}(\mathbf{x}), \pi^{kl}(\mathbf{y})\right\} &= \frac{1}{2}(\delta_i^k\delta_j^l + \delta_i^l\delta_j^k)\delta(\mathbf{x}-\mathbf{y}) \ , \quad \{\phi^A(\mathbf{x}), p_B(\mathbf{y})\} = \delta^A_B\delta(\mathbf{x}-\mathbf{y}) \ , \\
\{N(\mathbf{x}), \pi_N(\mathbf{y})\} &= \delta(\mathbf{x}-\mathbf{y}) \ , \quad \left\{\tilde{n}^i(\mathbf{x}), \pi_j(\mathbf{y})\right\} = \delta^i_j\delta(\mathbf{x}-\mathbf{y}) \ .
\end{aligned} \tag{2.35}$$

Now we show that the smeared form of the constraint $\tilde{\Sigma}_i \approx 0$

$$\mathbf{T}_S(\zeta^i) = \int d^3\mathbf{x} \zeta^i \tilde{\Sigma}_i \tag{2.36}$$

is the generator of the spatial diffeomorphism. First of all using (2.35) we find

$$\left\{\mathbf{T}_S(\zeta^i), \tilde{n}^k\right\} = -\zeta^i \partial_i \tilde{n}^k + \tilde{n}^j \partial_j \zeta^k \tag{2.37}$$

which is the correct transformation rule for $\tilde{n}^i$. Then using (2.35) we find

$$\begin{aligned}
\left\{\mathbf{T}_S(N^i), \mathcal{R}_j\right\} &= -\partial_i N^i \mathcal{R}_j - N^i \partial_i \mathcal{R}_j - \mathcal{R}_i \partial_j N^i \ , \\
\left\{\mathbf{T}_S(N^i), p_A\right\} &= -N^i \partial_i p_A - \partial_i N^i p_A \ , \\
\left\{\mathbf{T}_S(N^i), \phi^A\right\} &= -N^i \partial_i \phi^A \ , \\
\left\{\mathbf{T}_S(N^i), g_{ij}\right\} &= -N^k \partial_k g_{ij} - \partial_i N^k g_{kj} - g_{ik} \partial_j N^k \ , \\
\left\{\mathbf{T}_S(N^i), \pi^{ij}\right\} &= -\partial_k (N^k \pi^{ij}) + \partial_k N^i \pi^{kj} + \pi^{ik} \partial_k N^j \ , \\
\left\{\mathbf{T}_S(N^i), f_{ij}\right\} &= -N^k \partial_k f_{ij} - \partial_i N^k f_{kj} - f_{ik} \partial_j N^k \ , \\
\left\{\mathbf{T}_S(N^i), \pi^i\right\} &= -\partial_i N^i \pi^j - N^i \partial_i \pi^j + \partial_j N^i \pi^j \ ,
\end{aligned} \tag{2.38}$$

that are the correct transformation rules of the canonical variables under spatial diffeomorphism. To proceed further we need the Poisson bracket between $\mathbf{T}_S(N^i)$ and $\tilde{D}^i{}_j$. It turns out that it is convenient to know the explicit form of the matrix $\tilde{D}^i{}_j$ [37, 38, 41, 42]

$$\tilde{D}^i{}_j = \sqrt{g^{im} f_{mn} Q^n{}_p} (Q^{-1})^p{}_j \ , \tag{2.39}$$

where

$$Q^i{}_j = \tilde{x}\delta^i{}_j + \tilde{n}^i \tilde{n}^k f_{kj} \ , \quad (Q^{-1})^p{}_q = \frac{1}{\tilde{x}}(\delta^p{}_q - \tilde{n}^p \tilde{n}^m f_{mq}) \ . \tag{2.40}$$

Using this expression we can easily determine the Poisson brackets between $\mathbf{T}_S(N^i)$ and $\tilde{D}^k{}_j$. In fact, by definition we have

$$\begin{aligned}
\left\{\mathbf{T}_S(N^i), Q^k{}_l\right\} &= -N^m\partial_m Q^k{}_l + \partial_m N^k Q^m_l - Q^k{}_n\partial_l N^n \ , \\
\left\{\mathbf{T}_S(N^i), Q^i{}_j Q^j{}_k\right\} &= -N^m\partial_m (Q^i{}_j Q^j{}_k) + \partial_m N^i Q^i{}_j Q^j{}_k - Q^i{}_j Q^j{}_m \partial_k N^m \ .
\end{aligned} \tag{2.41}$$

Using (2.39) and the results derived above we find

$$\left\{\mathbf{T}_S(N^i), \tilde{D}^i{}_j\right\} = -N^m\partial_m \tilde{D}^i{}_j + \partial_m N^i \tilde{D}^m_j - \tilde{D}^i{}_m \partial_j N^m \ . \tag{2.42}$$

Collecting all these results and after some calculations we find

$$\begin{aligned}
\left\{\mathbf{T}_S(N^i), \mathcal{C}_0\right\} &= -N^m\partial_m\mathcal{C}_0 - \partial_m N^m \mathcal{C}_0 \ , \\
\left\{\mathbf{T}_S(N^i), \Sigma_p\right\} &= -N^m\partial_m\Sigma_p - \partial_m N^m \Sigma_p \ .
\end{aligned} \tag{2.43}$$

Note also that it is easy to show that following Poisson bracket holds

$$\left\{\mathbf{T}_S(N^i), \mathbf{T}_S(M^j)\right\} = \mathbf{T}_S(N^j\partial_j M^i - M^j\partial_j N^i) \ . \tag{2.44}$$

Now we are ready to analyze the stability of all primary constraints. As usual the requirement of the preservation of the constraint $\pi_N \approx 0$ implies an existence of the secondary constraint $\mathcal{C}_0 \approx 0$. However the fact that $\mathcal{C}_0$ is the constraint immediately implies that the constraint $\tilde{\Sigma}_i \approx 0$ is preserved during the time evolution of the system, using (2.43) and (2.44). Now we analyze the requirement of the preservation of the constraints $\pi_i \approx 0$ during the time evolution of the system

$$\partial_t \pi_i = \{\pi_i, H_E\} = -\left(\Omega_p \delta_i^k + \frac{\partial(\tilde{D}^k{}_j \tilde{n}^j)}{\partial \tilde{n}^i}\right)\left(\mathcal{R}_k - 2m^2 M_p^2 \frac{\sqrt{g}}{\sqrt{\tilde{x}}} f_{km}\tilde{n}^m\right) = 0 \ . \tag{2.45}$$

It turns out that the following matrix

$$\Omega_p \delta_i^k + \frac{\partial(\tilde{D}^k{}_j \tilde{n}^j)}{\partial \tilde{n}^i} = 0 \tag{2.46}$$

cannot be solved for Ω_p and hence we have to demand the existence of following secondary constraints [37, 38, 41, 42]

$$\mathcal{C}_i \equiv \mathcal{R}_i - \frac{2m^2 M_p^2 \sqrt{g}}{\sqrt{\tilde{x}}} f_{ij}\tilde{n}^j \approx 0 \ . \tag{2.47}$$

Finally we have to proceed to the analysis of the time development of the constraint $\Sigma_p \approx 0$. However it turns out that it is very difficult to perform this analysis for Σ_p due to the presence of the terms that contain the spatial derivatives of ϕ^A. Then the explicit

calculation gives $\{\Sigma_p(\mathbf{x}), \Sigma_p(\mathbf{y})\} \neq 0$. For that reason we proceed in a different way when we try to simplify the constraint Σ_p. Using $\mathcal{C}_i$ and Σ_i we find that

$$A = \frac{4m^4 M_p^4 g \tilde{n}^i f_{ij} \tilde{n}^j}{\sqrt{\tilde{x}}} + F^i \Sigma_i + G^i \mathcal{C}_i \ , \tag{2.48}$$

where

$$A = p_A \partial_i \phi^A f^{ij} \partial_j \phi^B p_B \ , \tag{2.49}$$

and where F^i, G^i are the phase space functions whose explicit form is not important for us. Then with the help of (2.48) we express $\tilde{n}^i f_{ij} \tilde{n}^j$ as a function of the phase space variables p_A, ϕ^A and g_{ij}, π^{ij}

$$\tilde{n}^i f_{ij} \tilde{n}^j = \frac{A - F^i \Sigma_i - G^i \mathcal{C}_i}{(A - F^i \Sigma_i - G^i \mathcal{C}_i) + 4m^4 M_p^4 g} \ . \tag{2.50}$$

In the same way we obtain

$$\tilde{n}^i \mathcal{R}_i = \frac{A - F^i \Sigma_i - G^i \mathcal{C}_i}{\sqrt{(A - F^i \Sigma_i - G^i \mathcal{C}_i) + 4m^4 M_p^4 g}} + \tilde{n}^i \mathcal{C}_i \ , \tag{2.51}$$

and

$$\tilde{n}^i = -\frac{\partial_j \phi^A p_A f^{ji}}{\sqrt{A + 4m^4 M_p^4 g}} + \tilde{F}^i \Sigma_i + \tilde{G}^i \mathcal{C}_i \ , \tag{2.52}$$

where again $\tilde{F}^i, \tilde{G}^i$ are phase space functions whose explicit form is not needed for us.

Now using these results we find that the constraint Σ_p takes the form

$$\begin{aligned} \Sigma_p &= \frac{(A - F^i \Sigma_i - G^i \mathcal{C}_i + 4m^4 M_p^4 g) 4m^4 M_p^4 g}{A - F^i \Sigma_i - G^i \mathcal{C}_i + 4m^4 M_p^4 g} + H^i \Sigma_i + p_A p^A = \\ &= p_A p^A + 4m^4 M_p^4 g + H^i \Sigma_i \equiv 4m^4 M_p^4 g \tilde{\Sigma}_p + H^i \Sigma_i \ , \end{aligned} \tag{2.53}$$

where we introduced new independent constraint $\tilde{\Sigma}_p$

$$\tilde{\Sigma}_p = \frac{p_A p^A}{4m^4 M_p^4 g} + 1 = 0 \tag{2.54}$$

that has precisely the same form as in [51]. Note that the constraint $\tilde{\Sigma}_p$ has the desired property that $\left\{\tilde{\Sigma}_p(\mathbf{x}), \tilde{\Sigma}_p(\mathbf{y})\right\} = 0$. As a result we see that it is more natural to consider $\tilde{\Sigma}_p$ instead of Σ_p as an independent constraint. Then the total Hamiltonian, where we include all constraints, takes the form

$$H_T = \int d^3\mathbf{x} (N \mathcal{C}_0 + v_N \pi_N + v^i \pi_i + \Omega_p \tilde{\Sigma}_p + \Omega^i \tilde{\Sigma}_i + \Gamma^i \mathcal{C}_i) \ . \tag{2.55}$$

Now we are ready to analyze the stability of all constraints that appear in (2.55). First of all we find that $\pi_N \approx 0$ is automatically preserved while the preservation of the constraint $\pi_i \approx 0$ gives

$$\begin{aligned} \partial_t \pi_i &= \{\pi_i, H_T\} \approx \int d^3\mathbf{x} \Gamma^j(\mathbf{x}) \{\pi_i, \mathcal{C}_j(\mathbf{x})\} = \\ &= -2m^2 \Gamma^j \frac{1}{\sqrt{\tilde{x}}}(f_{ij} - f_{ik}\tilde{n}^k f_{jl}\tilde{n}^l) \equiv -\triangle_{\pi_i,\mathcal{C}_j}\Gamma^j \ . \end{aligned} \tag{2.56}$$

By definition

$$\det(f_{ij} - f_{ik}\tilde{n}^k f_{il}\tilde{n}^l) = \tilde{x} \det f_{ij} \neq 0 \tag{2.57}$$

and hence the matrix $\triangle_{\pi_i,\mathcal{C}_j}$ is non-singular. Then the only solution of the equation (2.56) is $\Gamma^i = 0$.

As the next step we proceed to the analysis of the stability of the constraint $\tilde{\Sigma}_p$. As is clear from (2.54) we have

$$\left\{\tilde{\Sigma}_p(\mathbf{x}), \tilde{\Sigma}_p(\mathbf{y})\right\} = 0 \ . \tag{2.58}$$

Then the time evolution of given constraint takes the form

$$\partial_t \tilde{\Sigma}_p = \left\{\tilde{\Sigma}_p, H_T\right\} \approx \int d^3\mathbf{x} N(\mathbf{x}) \{\Sigma_p, \mathcal{C}_0(\mathbf{x})\} \tag{2.59}$$

using the fact that $\tilde{\Sigma}_p$ does not depend on $\tilde{n}^i$ together with $\Gamma^i = 0$ and also the fact that $\tilde{\Sigma}_p$ is manifestly diffeomorphism invariant.

In order to explicitly determine (2.59) we need following expression

$$\frac{\delta(\sqrt{\tilde{x}}\tilde{D}^k{}_k)}{\delta f_{ij}} = \frac{\sqrt{\tilde{x}}}{2}\tilde{D}^j{}_p f^{pi} - \frac{1}{\sqrt{\tilde{x}}}\tilde{n}^l f_{lm} \frac{\delta(\tilde{D}^m{}_p \tilde{n}^p)}{\delta f_{ij}} \ . \tag{2.60}$$

Then after some calculations we obtain

$$\begin{aligned} &\left\{\tilde{\Sigma}_p, \int d^3\mathbf{x} N \mathcal{C}_0\right\} = 2\partial_i[N\tilde{D}^i{}_j]\tilde{n}^j \Sigma_p + \\ &+ \frac{1}{M_p^4 m^4 g} p_A \partial_i \left[N \frac{\delta(\tilde{D}^k{}_l \tilde{n}^l)}{\delta f_{ij}} \mathcal{C}_k \partial_j \phi^A\right] - \frac{1}{M_p^4 m^4 g} p_A \partial_j \phi^A \partial_i \left[N \frac{\delta(\tilde{D}^k{}_l \tilde{n}^l)}{\delta f_{ij}}\right] \Sigma_k + \\ &+ N\left(-\tilde{D}^i{}_j \frac{\partial_i[\tilde{n}^j p_A] p_A}{2m_p^4 m^4 g} + \frac{2m^2 M_p^2}{M_p^4 m^4 g} \tilde{D}^i{}_k p_A \partial_i[\sqrt{g}\sqrt{\tilde{x}} f^{kj} \partial_j \phi^A]\right) \approx N\Sigma_p^{II} \ . \end{aligned} \tag{2.61}$$

In order to simplify Σ_p^{II} further we use (2.50), (2.51) together with (2.52) to make Σ_p^{II} independent on $\tilde{n}^i$. In fact, using (2.39)(2.40) we obtain

$$\begin{aligned}
Q_p^m &= \frac{1}{A + 4M_p^4 m^4 g}(4m^4 M_p^4 g \delta^m_{\ p} + \partial_j\phi^A p_A f^{jm}\partial_p\phi^B p_B) \ , \\
(Q^{-1})_p^m &= \frac{A + 4M_p^4 m^4 g}{4m^4 M_p^4 g}(\delta^m_{\ p} - \frac{1}{A + 4m^4 M_p^4 g}\partial_j\phi^A p_A f^{jm}\partial_p\phi^B p_B)
\end{aligned} \tag{2.62}$$

up to terms proportional to the constraints $\mathcal{C}_i, \Sigma_i$. With the help of these results we obtain

$$\begin{aligned}
\Sigma_p^{II} &= -\tilde{D}^i_{\ j}\frac{\partial_i[\tilde{n}^j p_A]p_A}{2m_p^4 m^4 g} + \frac{2m^2 M_p^2}{M_p^4 m^4 g}\tilde{D}^i_{\ p} p_A \partial_i[\sqrt{g}\sqrt{\tilde{x}} f^{pj}\partial_j\phi^A] + F'^i\Sigma_i + G'^i\mathcal{C}_i \equiv \\
&\equiv \tilde{\Sigma}_p^{II} + F'^i\Sigma_i + G'^i\mathcal{C}_i \ ,
\end{aligned} \tag{2.63}$$

where $\tilde{x}$ and $\tilde{D}^i_{\ j}$ are functions of $p_A, \partial_j\phi^A$ and g through the relations (2.50),(2.51) and (2.52). Now we see from (2.59) that the time evolution of the constraint $\tilde{\Sigma}_p \approx 0$ is obeyed on condition when either $N = 0$ or when $\tilde{\Sigma}_p^{II} = 0$. Note that we should interpreted N as the Lagrange multiplier so that it is possible to demand that $N = 0$ on condition when $\tilde{\Sigma}_p^{II} \neq 0$ on the whole phase space. Of course such a condition is too strong so that it is more natural to demand that $\tilde{\Sigma}_p^{II} \approx 0$ and $N \neq 0$. In other words $\tilde{\Sigma}_p^{II} \approx 0$ is the new secondary constraint.

In summary we have following collection of constraints: $\pi_N \approx 0$, $\pi_i \approx 0$, $\mathcal{C}_0 \approx 0, \mathcal{C}_i \approx 0, \tilde{\Sigma}_i \approx 0, \tilde{\Sigma}_p \approx 0, \tilde{\Sigma}_p^{II} \approx 0$. The dynamics of these constraints is governed by the total Hamiltonian

$$H_T = \int d^3\mathbf{x}(N\mathcal{C}_0 + v_N\pi_N + v^i\pi_i + \Omega_p\tilde{\Sigma}_p + \Omega_p^{II}\tilde{\Sigma}_p^{II} + \Omega^i\tilde{\Sigma}_i + \Gamma^i\mathcal{C}_i) \ . \tag{2.64}$$

As the final step we have to analyze the preservation of all constraints. The case of $\pi_N \approx 0$ is trivial. For $\pi_i \approx 0$ we obtain

$$\begin{aligned}
\partial_i\pi_i(\mathbf{x}) &= \{\pi_i(\mathbf{x}), H_T\} = \int d^3\mathbf{y}(\Gamma^j(\mathbf{y})\{\pi_i(\mathbf{x}), \mathcal{C}_j(\mathbf{y})\} + \Omega_p^{II}(\mathbf{y})\left\{\pi_i(\mathbf{x}), \tilde{\Sigma}_p^{II}(\mathbf{y})\right\}) = \\
&= \Gamma^j \triangle_{\pi_i,\mathcal{C}_j}(\mathbf{x}) = 0
\end{aligned} \tag{2.65}$$

due to the crucial fact that $\tilde{\Sigma}_p^{II}$ does not depend on $\tilde{n}^i$. This is the main reason why we introduced $\tilde{\Sigma}_p^{II}$ instead of Σ_p^{II}. Then as we argued above the only solution of the equation is $\Gamma^i = 0$. Now the time development of $\mathcal{C}_i$ is given by the equation

$$\begin{aligned}
\partial_t\mathcal{C}_i(\mathbf{x}) &= \{\mathcal{C}_i(\mathbf{x}), H_T\} \approx \\
&\approx \int d^3\mathbf{x}\left(N(\mathbf{y})\{\mathcal{C}_i(\mathbf{x}), \mathcal{C}_0(\mathbf{y})\} + v^j(\mathbf{y})\{\mathcal{C}_i(\mathbf{x}), \pi_j(\mathbf{y})\} + \right. \\
&+ \left. \Omega_p(\mathbf{y})\left\{\mathcal{C}_i(\mathbf{x}), \tilde{\Sigma}_p(\mathbf{y})\right\} + \Omega_p^{II}(\mathbf{y})\left\{\mathcal{C}_i(\mathbf{x}), \tilde{\Sigma}_p^{II}(\mathbf{y})\right\}\right)
\end{aligned} \tag{2.66}$$

and the time development of the constraint $\tilde{\Sigma}_p$ is governed by the equation

$$\partial_t \tilde{\Sigma}_p(\mathbf{x}) = \left\{\tilde{\Sigma}_p(\mathbf{x}), H_T\right\} \approx \int d^3\mathbf{x}\Omega_p^{II}(\mathbf{y}) \left\{\tilde{\Sigma}_p(\mathbf{x}), \tilde{\Sigma}_p^{II}(\mathbf{y})\right\} \ . \tag{2.67}$$

As follows from the explicit form of the constraint $\tilde{\Sigma}_p^{II}$ we see that $\left\{\tilde{\Sigma}_p^{II}(\mathbf{x}), \tilde{\Sigma}_p(\mathbf{y})\right\}$ is non-zero and proportional also to the higher order derivatives of the delta functions. As a consequence we find that the only solution of the equation above is $\Omega_p^{II} = 0$. Further we analyze the time evolution of the constraint $\tilde{\Sigma}_p^{II}$

$$\begin{aligned}
\partial_t \tilde{\Sigma}_p^{II}(\mathbf{x}) &= \left\{\tilde{\Sigma}_p^{II}(\mathbf{x}), H_T\right\} = \\
&= \int d^3\mathbf{x} \left(N(\mathbf{y}) \left\{\tilde{\Sigma}_p^{II}(\mathbf{x}), \mathcal{C}_0(\mathbf{y})\right\} + \Omega_p(\mathbf{y}) \left\{\tilde{\Sigma}_p^{II}(\mathbf{x}), \tilde{\Sigma}_p(\mathbf{y})\right\}\right) = 0 \ .
\end{aligned} \tag{2.68}$$

Now from the last equation we obtain Ω_p as a function of the phase space variables and N, at least in principle. Then inserting this result into the equation for the preservation of $\mathcal{C}_i$ (2.66) we determine v^j as functions of the phase space variables. Finally note also that the constraint $\mathcal{C}_0$ is automatically preserved due to the fact that $\Gamma^i = \Omega_p^{II} = 0$ and also the fact that $\{\mathcal{C}_0(\mathbf{x}), \mathcal{C}_0(\mathbf{y})\} \approx 0$ as was shown in [40].

In summary we obtain following picture. We have five the first class constraints $\pi_N \approx 0 \ , \mathcal{C}_0 \approx 0 \ , \tilde{\Sigma}_i \approx 0$ together with eight the second class constraints $\pi_i \approx 0 \ , \mathcal{C}_i \approx 0$ and $\tilde{\Sigma}_p \approx 0 \ , \tilde{\Sigma}_p^{II} \approx 0$. The constraints $\pi_i \approx 0$ together with $\mathcal{C}_i \approx 0$ can be solved for π_i and $\tilde{n}^i$. Then the constraint $\tilde{\Sigma}_p$ can be solved for one of the four momenta p_A while the constraint $\tilde{\Sigma}_p^{II}$ can be solved for one of the four ϕ^A. As a result we have 12 gravitational degrees of freedom g_{ij}, π^{ij},6 scalars degrees of freedom together with 4 first class constraints $\mathcal{C}_0 \approx 0 \ , \tilde{\Sigma}_i \approx 0$. Then we find that the number of physical degrees of freedom is 10 which is the correct number of physical degrees of freedom of the massive gravity.

3. General Non-Linear Massive Gravity Action

Let us try to apply the procedure performed in previous section to the case of the general non-linear massive gravity whose action takes the form

$$S = M_p^2 \int d^3\mathbf{x} dt [N\sqrt{g}\tilde{K}_{ij}\mathcal{G}^{ijkl}\tilde{K}_{kl} + \sqrt{g}NR + 2m^2\sqrt{g}MU + 2m^2N\sqrt{g}V] \ , \tag{3.1}$$

where

$$\begin{aligned}
U &= \beta_1\sqrt{\tilde{x}} + \beta_2[(\sqrt{\tilde{x}})^2\tilde{D}^i{}_i + \tilde{n}^i f_{ij}\tilde{D}^j{}_k\tilde{n}^k] + \\
&+ \beta_3[\sqrt{\tilde{x}}(\tilde{D}^l{}_l\tilde{n}^i f_{ij}\tilde{D}^j{}_k\tilde{n}^k - \tilde{D}^i{}_k\tilde{n}^k f_{ij}\tilde{D}^j_l\tilde{n}^l) + \frac{1}{2}\sqrt{\tilde{x}}^3(\tilde{D}^i{}_i\tilde{D}^j{}_j - \tilde{D}^i{}_j\tilde{D}^j{}_i)] \ , \\
V &= \beta_0 + \beta_1\sqrt{\tilde{x}}\tilde{D}^i{}_i + \frac{1}{2}\sqrt{\tilde{x}}^2[\tilde{D}^i{}_i\tilde{D}^j{}_j + \tilde{D}^i{}_j\tilde{D}^j{}_i] + \\
&+ \frac{1}{6}\beta_3\sqrt{\tilde{x}}^3[\tilde{D}^i{}_i\tilde{D}^j{}_j\tilde{D}^k{}_k - 3\tilde{D}^i{}_i\tilde{D}^j{}_k\tilde{D}^k{}_j + 2\tilde{D}^i{}_j\tilde{D}^j{}_k\tilde{D}^k{}_i]
\end{aligned} \tag{3.2}$$

Following the analysis performed in the previous section we find the extended Hamiltonian in the form

$$H_E = \int d^3\mathbf{x}(N\mathcal{C}_0 + v_N\pi_N + v^i\pi_i + \Omega_p\Sigma_p + \Omega^i\tilde{\Sigma}_i) \ , \qquad (3.3)$$

where

$$\mathcal{C}_0 = \frac{1}{\sqrt{g}M_p^2}\pi^{ij}\mathcal{G}_{ijkl}\pi^{kl} - M_p^2\sqrt{g}R - 2m^2M_p^2V + \mathcal{R}_i\tilde{D}^i{}_j\tilde{n}^j \ , \qquad (3.4)$$

and where the primary constraint Σ_p takes the form

$$\Sigma_p : (\tilde{n}^i\mathcal{R}_i + 2m^2M_p^2\sqrt{g}U)^2 + p_A\mathcal{M}^{AB}p_B \approx 0 \ . \qquad (3.5)$$

As the next step we should analyze the stability of all primary constraints. As in previous section we find that the stability of the constraint $\pi_N \approx 0$ implies the secondary constraint $\mathcal{C}_0 \approx 0$ while the stability of the constraints π_i implies set of the secondary constraints $\mathcal{C}_i$ [37, 38, 41, 42]

$$\begin{gathered}\mathcal{C}_i = \mathcal{R}_i - 2m^2\sqrt{g}\frac{\tilde{n}^l f_{lj}}{\sqrt{\tilde{x}}}\left[\beta_1\delta_i^j + \beta_2\sqrt{\tilde{x}}(\delta_i^j\tilde{D}^m_{\ m} - \tilde{D}^j_{\ i}) + \right. \\ \left. +\beta_3\sqrt{\tilde{x}}^2\left(\frac{1}{2}\delta^j{}_i(\tilde{D}^m_{\ m}\tilde{D}^n_{\ n} - \tilde{D}^m_{\ n}\tilde{D}^n_{\ m}) + \tilde{D}^j{}_m\tilde{D}^m_i - \tilde{D}^j{}_i\tilde{D}^m_{\ m}\right)\right] \ .\end{gathered} \qquad (3.6)$$

Now we come to the key point of the analysis which is the requirement of the preservation of the constraint $\Sigma_p \approx 0$ during the time evolution of the system. This is very complicated expression which depends on the all phase space variables. Remember that in the minimal case we expressed $\tilde{n}^i$ as functions of g_{ij}, p_A and ϕ^A. As a result we found that Σ_p can be expressed as a linear combination of $\mathcal{C}_i, \Sigma_i$ and $\tilde{\Sigma}_p$ where $\tilde{\Sigma}_p$ obeys an important property $\left\{\tilde{\Sigma}_p(\mathbf{x}), \tilde{\Sigma}_p(\mathbf{y})\right\} = 0$.It would be certainly nice to repeat the same procedure in the case of the constraint Σ_p given in (3.5). Unfortunately we are not able to solve the constraint $\mathcal{C}_i$ in order to express $\tilde{n}^i$ as a function of $\mathcal{R}_i$. Consequently we are not able to express the constraint Σ_p as a linear combination of the constraints $\mathcal{C}_i$ and possibly $\tilde{\Sigma}_i, \mathcal{C}_0$ and the new constraint $\tilde{\Sigma}_p$ where $\left\{\tilde{\Sigma}_p(\mathbf{x}), \tilde{\Sigma}_p(\mathbf{y})\right\} = 0$. In other words, despite of the fact we are able to find the primary constraint $\Sigma_p \approx 0$ we are not able to determine the additional secondary constraint which is necessary for the elimination of two non-physical phase space modes. As a result the main goal of this paper which was the proof of the absence of the ghosts in the general non-linear massive gravity action in Stückelberg formalism cannot be completed.

Acknowledgement: This work was supported by the Grant agency of the Czech republic under the grant P201/12/G028.

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